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Dynamic Characterization of Connections in Plane Frames using SFFEM

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Abstract

In the construction of civil engineering structures, two or more members are often rigidly connected to increase the structural integrity. These rigid joints are often designed with bolts, rivets and welding. The actions of in-service loading and environmental effects, or fabrication errors make these joints semirigid, which ultimately reduces the structural reliability. Realistic dynamic analysis of these structures requires accurate modelling of rigidity of joints. Dynamic analysis of plane frames can be accomplished by combining spectrally formulated Rod and Euler-Bernoulli Beam element. In this study, a six parameter spectral plane frame joint element is formulated using linear and rotational springs to account for semi-rigidity of joints. Methodology and experimental set up for evaluation of dynamic characteristics of connections is discussed in this paper.

Keywords: Connections; Frames; Spectral element method; Semirigid.

1. Introduction

In steel frame structures various types are connections are employed viz. single web, double web, bottom seat, top and bottom seat, stiffened seat connection etc. Also, the various types of connections for column base are used like slab base, gusseted base etc. The dynamic response of different connections is necessarily required to be

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accurately modeled so that exact dynamic stiffness of connections can be incorporated in dynamic analysis of frames. Spectral analysis of plane frames is illustrated by M. Martin et al. [1, 5] by combining formulation of spectral deep rod element and Timoshenko beam element. A four parameter spectral joint model for bolted connection in beams is presented by Usik Lee [2].

A six parameter spectral element joint model is formulated taking into account three DOFs at each node viz. u, v and θ for axial, translational and rotational displacement. The formulation is done in such a way that it can be used to represent any type of joint in plane frame structure. This joint element is proposed to be used along with a spectral frame element formulated by combining spectral elementary rod element and spectral Euler Bernoulli beam element. The details of the proposed element are shown in the figure (1).

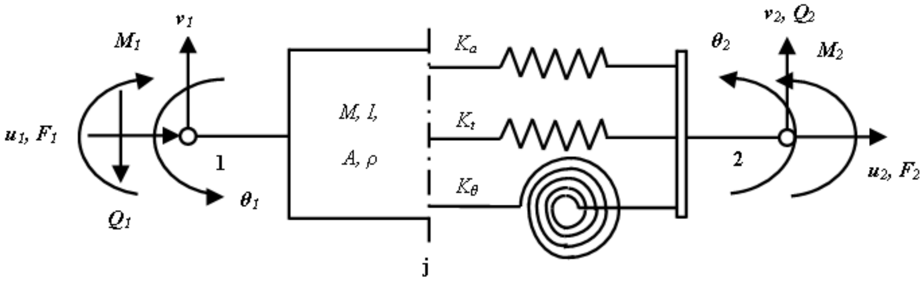

Fig. 1. Equivalent Six Parameter Joint Model

The formulation is done in such a way that it can be used to represent any type of joint in plane frame structure. This joint element is proposed to be used along with a spectral frame element formulated by combining spectral elementary rod element and spectral Euler Bernoulli beam element. The details of the proposed element are shown in the figure (1).

2. Methodology

2.1 Transfer Matrix for the Joint Model

The transfer matrix of the joint model is derived by using the sign conventions defined in figure (2). Using kinematic continuity and dynamic equilibrium conditions of the mass system and four pole techniques discussed by S. K. Clark [3], the transfer matrix equation of the mass system is derived as given by equation (1). The quantities F, Q, M are axial force, shear force and bending moment and u, v, θ are respective axial, transverse and rotational displacements. The quantities ρ, A, M, I are respectively density, area of cross section, mass and moment of inertia of the connection. The quantities subscripted by 1 are at node 1 and the quantities subscripted by j are at the junction between the inertia system and the spring system. The spring stiffnesses K_a, K_t, K_θ are axial, transverse and rotational stiffness of the connection.

$$\begin{Bmatrix} F_1 \\ u_1 \\ Q_1 \\ v_1 \\ M_1 \\ \theta_1 \end{Bmatrix} = \begin{bmatrix} 1 & \rho A \omega^2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & M\omega^2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & I\omega^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} F_j \\ u_j \\ Q_j \\ v_j \\ M_j \\ \theta_j \end{Bmatrix} \tag{1}$$

Similarly, the transfer matrix equation for the spring system is obtained by using sign conventions defined in figure (3) which is as given by equation (2).

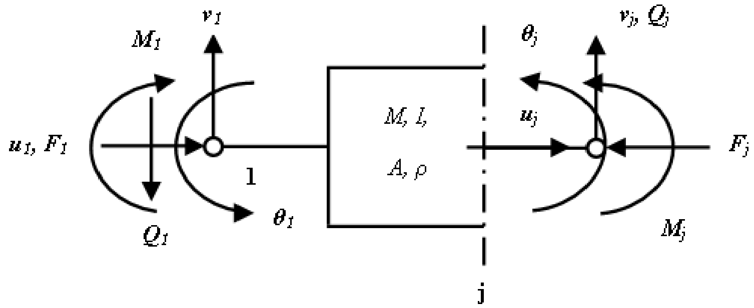

Fig. 2. Transfer Matrix for Mass System

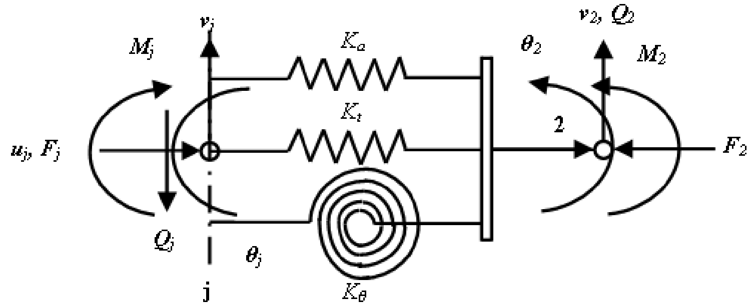

Fig. 3. Transfer Matrix for Spring System

$$\begin{Bmatrix} F_j \\ u_j \\ Q_j \\ v_j \\ M_j \\ \theta_j \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1/K_a & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1/K_t & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1/K_\theta & 1 \end{bmatrix} \begin{Bmatrix} F_2 \\ u_2 \\ Q_2 \\ v_2 \\ M_2 \\ \theta_2 \end{Bmatrix} \qquad (2)$$

By combining equations (2) and (3), a transfer equation relating nodal values of node 1 and node 2 is obtained as given by equation (3).

$$\begin{Bmatrix} F_2 \\ u_2 \\ Q_2 \\ v_2 \\ M_2 \\ \theta_2 \end{Bmatrix} = \begin{bmatrix} 1 & -\rho A\omega^2 & 0 & 0 & 0 & 0 \\ 1/K_a & 1-(\rho A\omega^2/K_a) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -M\omega^2 & 0 & 0 \\ 0 & 0 & 1/K_t & 1-(M\omega^2/K_t) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -I\omega^2 \\ 0 & 0 & 0 & 0 & 1/K_\theta & 1-(I\omega^2/K_\theta) \end{bmatrix} \begin{Bmatrix} F_1 \\ u_1 \\ Q_1 \\ v_1 \\ M_1 \\ \theta_1 \end{Bmatrix} \qquad (3)$$

The equation (3) is rearranged for displacements and forces for further simplicity as given by equation (4).

$$\begin{Bmatrix} u_2 \\ v_2 \\ \theta_2 \\ F_2 \\ Q_2 \\ M_2 \end{Bmatrix} = \begin{bmatrix} 1-(\rho A\omega^2/K_a) & 0 & 0 & 1/K_a & 0 & 0 \\ 0 & 1-(M\omega^2/K_t) & 0 & 0 & 1/K_t & 0 \\ 0 & 0 & 1-(I\omega^2/K_\theta) & 0 & 0 & 1/K_\theta \\ -\rho A\omega^2 & 0 & 0 & 1 & 0 & 0 \\ 0 & -M\omega^2 & 0 & 0 & 1 & 0 \\ 0 & 0 & -I\omega^2 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ F_1 \\ Q_1 \\ M_1 \end{Bmatrix} \tag{4}$$

Rearranging all displacements at right hand side and all forces at left hand side, the following force displacement relationship is obtained as given by equation (5).

$$\begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} \begin{bmatrix} -K_a+\rho A\omega^2 & 0 & 0 & K_a & 0 & 0 \\ 0 & -K_t+M\omega^2 & 0 & 0 & K_t & 0 \\ 0 & 0 & -K_\theta+I\omega^2 & 0 & 0 & K_\theta \\ -K_a & 0 & 0 & K_a & 0 & 0 \\ 0 & -K_t & 0 & 0 & K_t & 0 \\ 0 & 0 & -K_\theta & 0 & 0 & K_\theta \end{bmatrix} = \begin{Bmatrix} F_1 \\ Q_1 \\ M_1 \\ F_2 \\ V_2 \\ Q_2 \end{Bmatrix} \tag{5}$$

2.2 Equivalent Spectral Joint Element

The force displacement relationship given by equation (5) can be used directly for defining force displacement relationship for Equivalent Spectral Joint Element by giving due attention to the sign conventions adopted. The sign conventions used for forces in deriving transfer matrix and sign conventions for spectral element matrix are shown in figure (4) and figure (5). Hence by changing signs of the forces in equation (5), the spectral element relation can be obtained as given by equation (6).

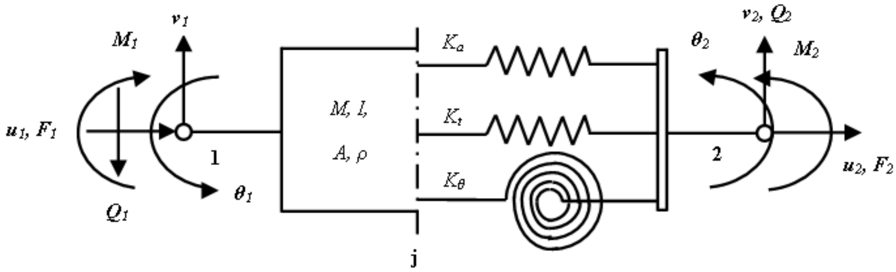

Fig. 4. Sign conventions defined for Transfer Matrix

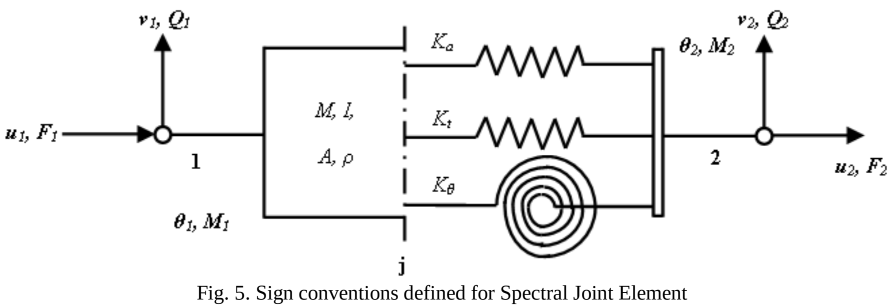

Fig. 5. Sign conventions defined for Spectral Joint Element

The spectral element equation for the joint part takes the form as,

$$K_j(\omega)d_j = f_j \tag{6}$$

where,

$$K_j(\omega) = \begin{bmatrix} K_a - \rho A\omega^2 & 0 & 0 & -K_a & 0 & 0 \\ 0 & K_t - M\omega^2 & 0 & 0 & -K_t & 0 \\ 0 & 0 & K_\theta - I\omega^2 & 0 & 0 & -K_\theta \\ -K_a & 0 & 0 & K_a & 0 & 0 \\ 0 & -K_t & 0 & 0 & K_t & 0 \\ 0 & 0 & -K_\theta & 0 & 0 & K_\theta \end{bmatrix} \tag{7}$$

and,

$$d_j = \{u_1 \quad v_1 \quad \theta_1 \quad u_2 \quad v_2 \quad \theta_2\}^T \tag{8}$$

$$f_j = \{F_1 \quad Q_1 \quad M_1 \quad F_2 \quad Q_2 \quad M_2\}^T \tag{9}$$

The frequency dependent symmetric matrix $K_j(\omega)$ is the spectral element matrix for the joint part of plane frame containing six unknown parameters ρA, M, I, K_a, K_t, K_θ which are required to be determined experimentally.

3. Proposed Experimental Set-Up and Program

It is proposed to measure the unknown parameters by conducting experiments on laboratory models of plane frames having different rigidities at the beam-column connections and beam-base connections. The detailed experimental set up is as shown in figure (6). In the experimental program, it is proposed to use fifteen spectral frame elements and four spectral joint elements. The total number of nodes is 21 raising 63 DOFs in the system. The measurement of displacements at nodes 1, 7, 8, 9, 14, 15 and 21 is difficult due to complexities of connection etc. Also, measurement of axial and transverse displacements is easy in comparison with measurements of rotational displacements. Thus, out of 21 nodes it is proposed to measure vertical displacements at 12 nodes and horizontal displacements at 12 nodes giving set of 24 measured displacements.

For the structure shown in figure (6), 63 equations will be available in global force displacement relation. The thirty nine unmeasured displacements and the six unknown joint parameters at each joint raise sixty three unknowns which can be solved by global stiffness equation of the system. Once the unknown frequency dependent joint parameters are obtained the dynamic analysis of plane frame with the type of semirigid joint under consideration will be performed under different loading conditions.

4. Conclusion and Discussion

Unmeasured displacements and unknown joint parameters can be obtained as discussed in this section. A global force displacement relation for experimental set up shown in figure (6) will be as given by,

$$K(\omega)\, d = f \tag{10}$$

where,

$$d = \{u_1 \quad v_1 \quad \theta_1 \quad u_2 \quad v_2 \quad \theta_2 \; \ldots\ldots\ldots\ldots \; u_{21} \quad v_{21} \quad \theta_{21}\}^T \tag{11}$$

$$f = \{0 \quad \ldots\ldots\ldots\ldots \quad F(\omega) \; \ldots\ldots\ldots\ldots \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\}^T \tag{12}$$

in which, $K(\omega)$ is global assembled stiffness matrix, $f_9 = F(\omega)$ and at all other nodes $f = 0$. Equation (10) can be written in partitioned form as,

$$\begin{bmatrix} K_{0m} & K_{0u} \\ K_{em} & K_{eu} \end{bmatrix} \begin{Bmatrix} D_m \\ D_u \end{Bmatrix} = \begin{Bmatrix} 0 \\ f_e \end{Bmatrix} \tag{13}$$

where, f_e will be 24 element column matrix containing $F(\omega)$ at respective node and all other elements to be zero. By usual procedure of static condensation the unmeasured displacements can be calculated from which knowing all displacements and forces unknown six joint parameters at each joint can be obtained. It is proposed to conduct experiments on laboratory models of plane frames with various types of beam column and column base connections and validate the proposed procedure.

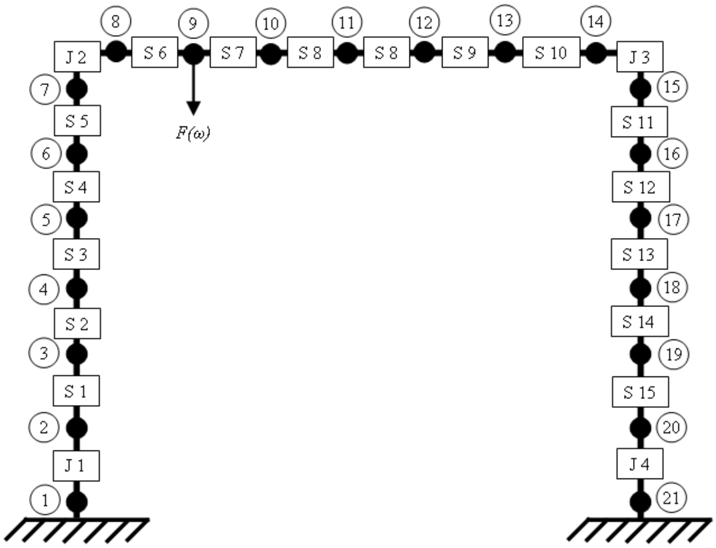

Fig. 6. Proposed Experimental Set-up & Details of Discretisation of Frame

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